

How to reconstruct vertical velocities?

Summary of results obtained by several groups:

- A. Pietri, F. D'ovidio and X. Capet (LOCEAN)
- B. Qiu (Hawaii)
- A. Ponte, P. Klein and P. Rivière (IFREMER)
- G. Lapeyre (LMD) and S. Berti (LML)

- Basically, two methods for reconstructing w :
 - "Full method": Solve an **omega-equation**
 - Deduced from primitive equations
 - Involves **3D** quantities: density and velocity field
 - "Simplified method": Solve a **SQG equation**
 - Strong assumption of SQG dynamics
 - Only involves **surface** quantities (SSH and/or density)
- *The first method will give better results but we need (u, v, ρ) at every depth!*

1st method: Vertical velocity from omega equation

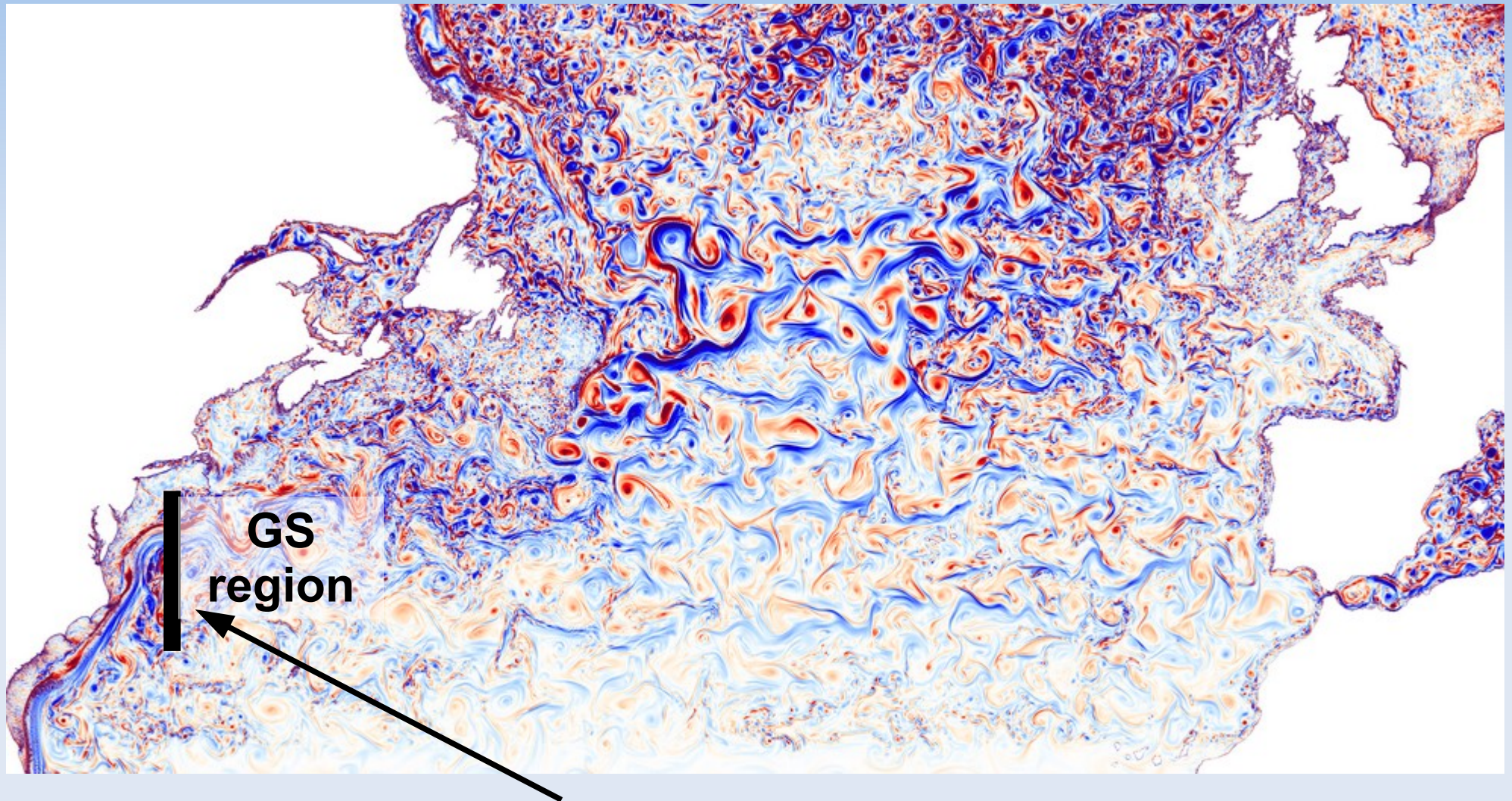
$$\nabla_H \cdot (N^2 \nabla_H w) + f_0^2 \frac{\partial^2 w}{\partial z^2} = -2 \nabla_H \cdot \mathbf{Q} + \underbrace{\text{term 1}}_{\text{ageostrophy}} + \underbrace{\text{term 2}}_{\text{surface fluxes}}$$

$$\mathbf{Q}(x, y, z) = \frac{2g}{\rho_0} [\nabla_H \mathbf{u}_H] \nabla_H \rho$$

➡ Inversion of a 3D elliptic equation. Needs

- 3D density field
- SSH for computation of horizontal velocity
- Surface fluxes (wind stress, buoyancy flux, vertical mixing)
- QG approximation with term 1 = term 2 = 0

North Atlantic simulation (1/60deg) (LGGE team, Grenoble)



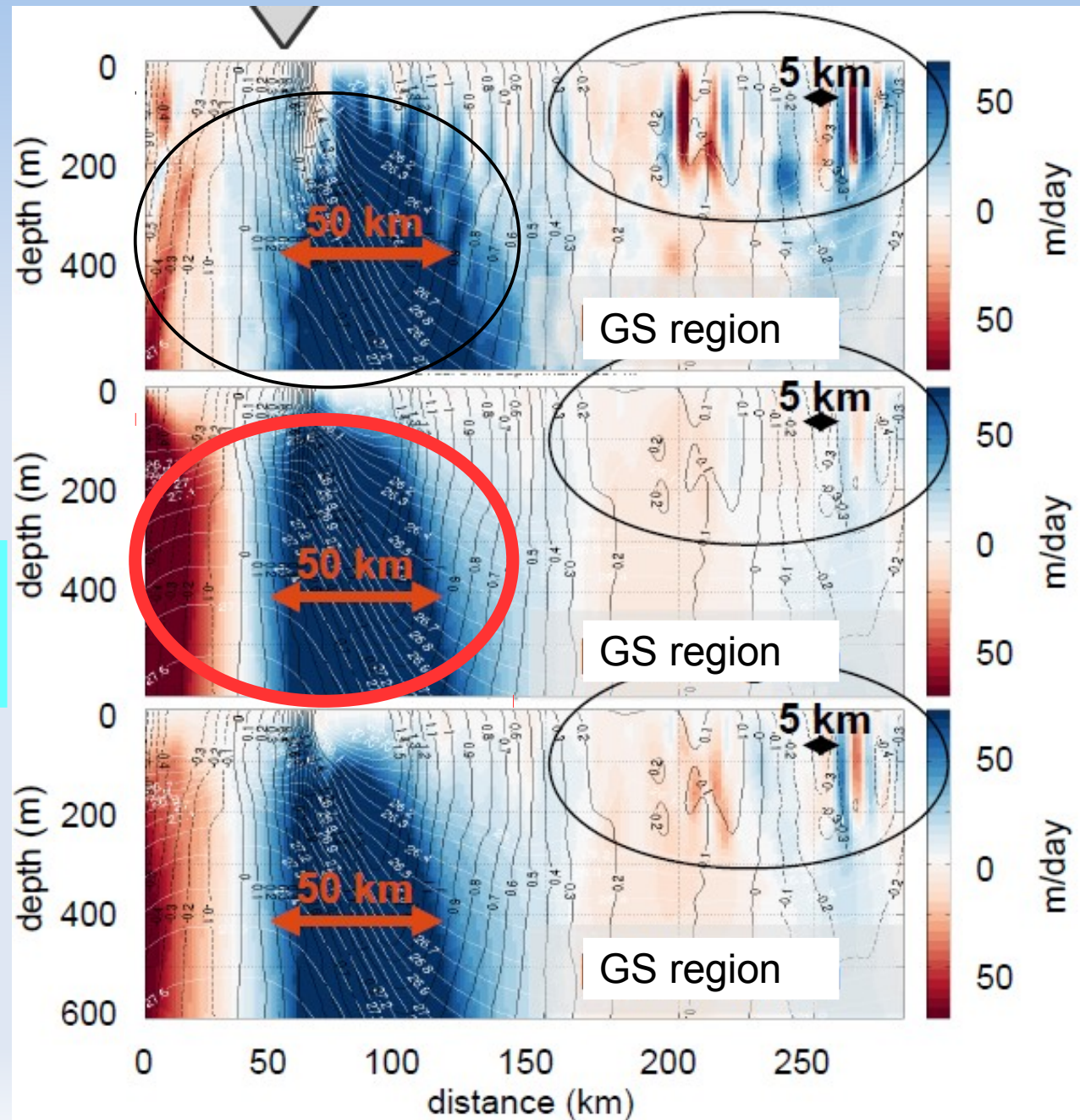
**Vertical cross section in order to look
at vertical velocities reconstruction**

Vertical velocity reconstruction in GS region (Pietri, Capet and D'ovidio)

W from model

W from Q vector
with QG terms only

Reconstruction of w at
mesoscales (50km)
and at depth

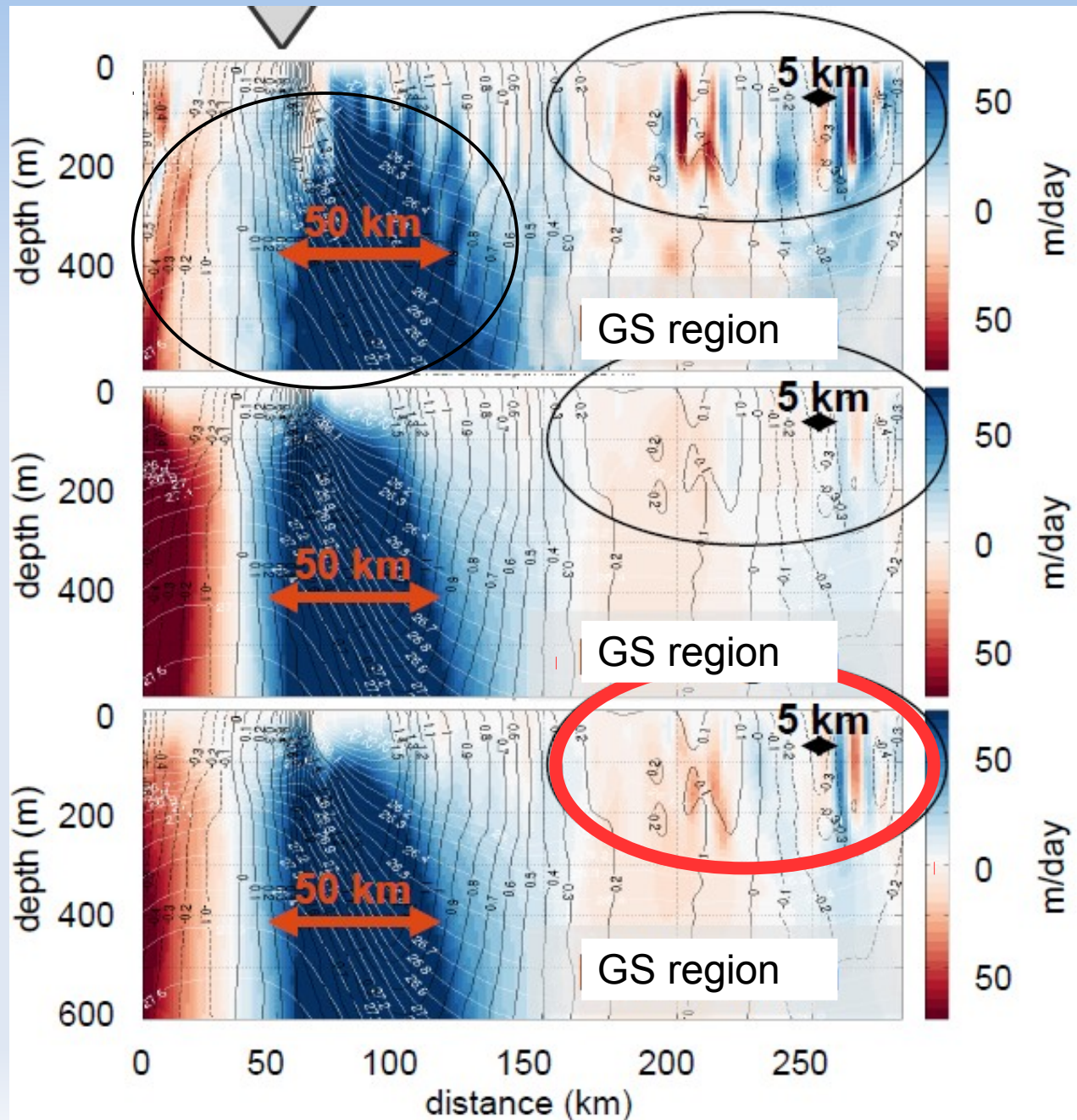


Vertical velocity reconstruction in GS region (Pietri, Capet and D'ovidio)

W from model

W from QG and
non-geostrophic terms
(e.g. surface fluxes)

Reconstruction of w at
submesoscales (5km)
and in the mixed layer



- Omega-equation using QG terms does a good job
 - In regions of very intense mesoscales (e.g. Gulf Stream, Kuroshio)
 - For scales larger than 60km and for deep motions
- Ageostrophic terms and surface fluxes improve the reconstruction
 - For scales larger than 30km and subsurface motions
 - In regions of weak mesoscales or strong surface fluxes (e.g. NorthEast Atlantic)
- Results depend on season and location (not shown)
- Next step: How does SWOT noise affect reconstruction?

2nd method: Vertical velocity from SQG theory

SQG= Assumption of no PV anomalies

$$PV = \nabla_H^2 \psi + \frac{\partial}{\partial z} \left(\frac{f_0^2}{N^2} \frac{\partial \psi}{\partial z} \right) = 0$$

with

$$\left. \frac{\partial \psi}{\partial z} \right|_{z=0} = -\frac{g}{f_0 \rho_0} \rho_s$$

Horizontal flow reconstruction:

$$\hat{\psi}(\mathbf{k}, z) = \frac{g}{f_0} \hat{\eta} \exp \left(\frac{N}{f_0} k z \right) = -\frac{g}{N \rho_0 k} \hat{\rho}_s \exp \left(\frac{N}{f_0} k z \right)$$

In horizontal Fourier space

with

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} -\partial_y \psi \\ \partial_x \psi \end{pmatrix}$$

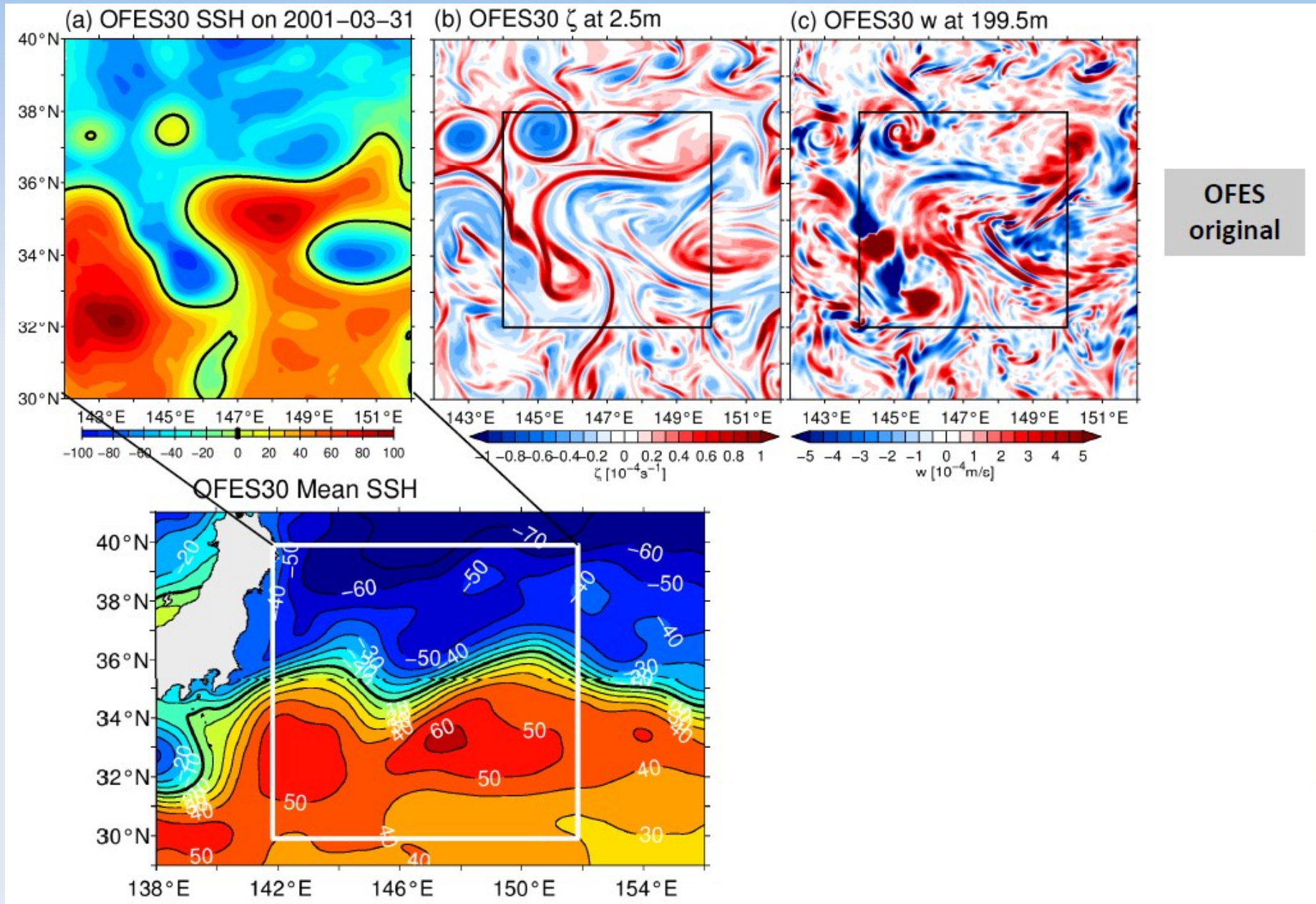
Vertical flow reconstruction:

$$\hat{w} = \frac{1}{N^2} \left(-J(\widehat{\psi_s}, \widehat{b_s}) \exp \left(\frac{N}{f_0} |k| z \right) + J(\widehat{\psi}, \widehat{b}) \right)$$

with

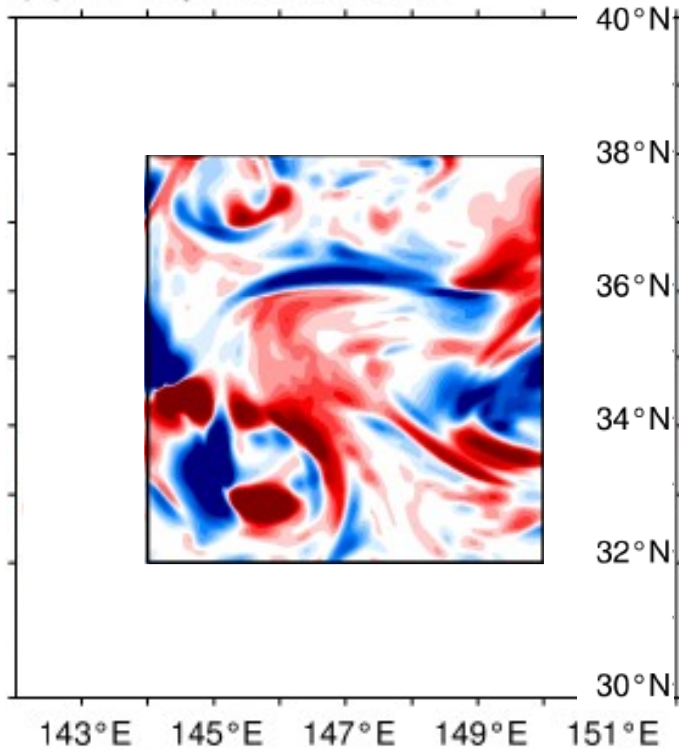
$$b = -\frac{g\rho}{\rho_0}$$

A specific example: Kuroshio extension region (Qiu et al., 2016, JPO)

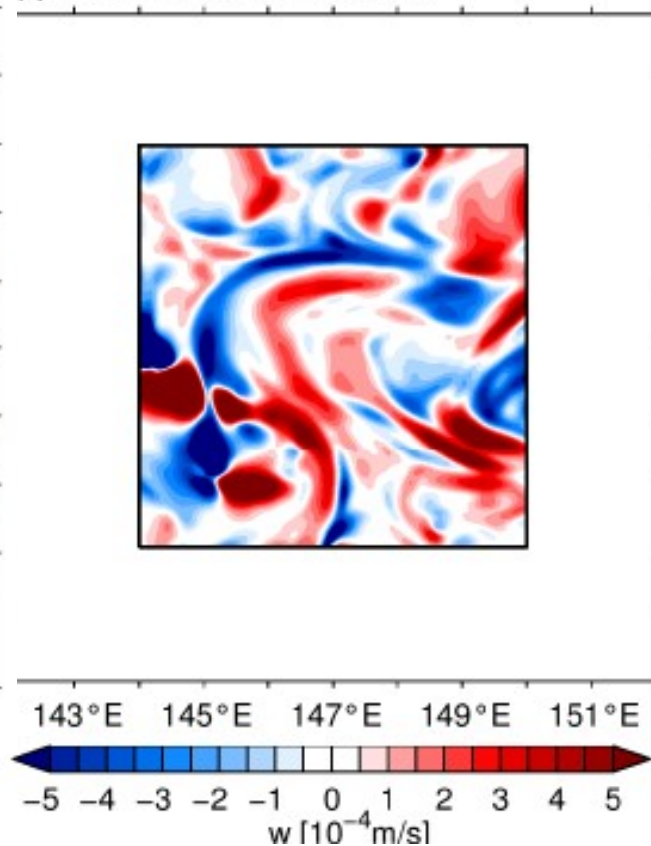


SQG reconstruction with modeled SSH and no SWOT errors

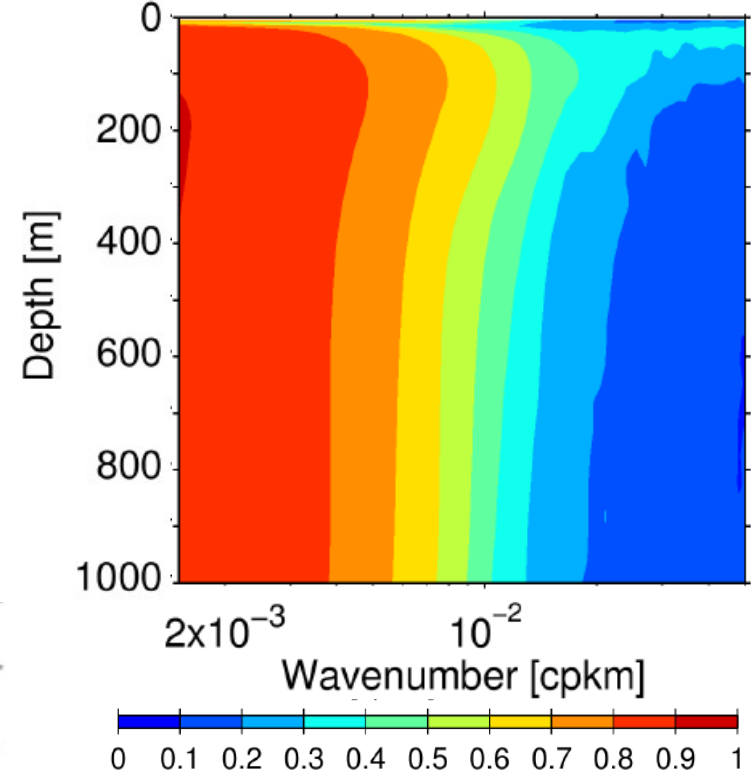
(c) Ω -eq w at 199.5m



(f) eSQG w at 199.5m

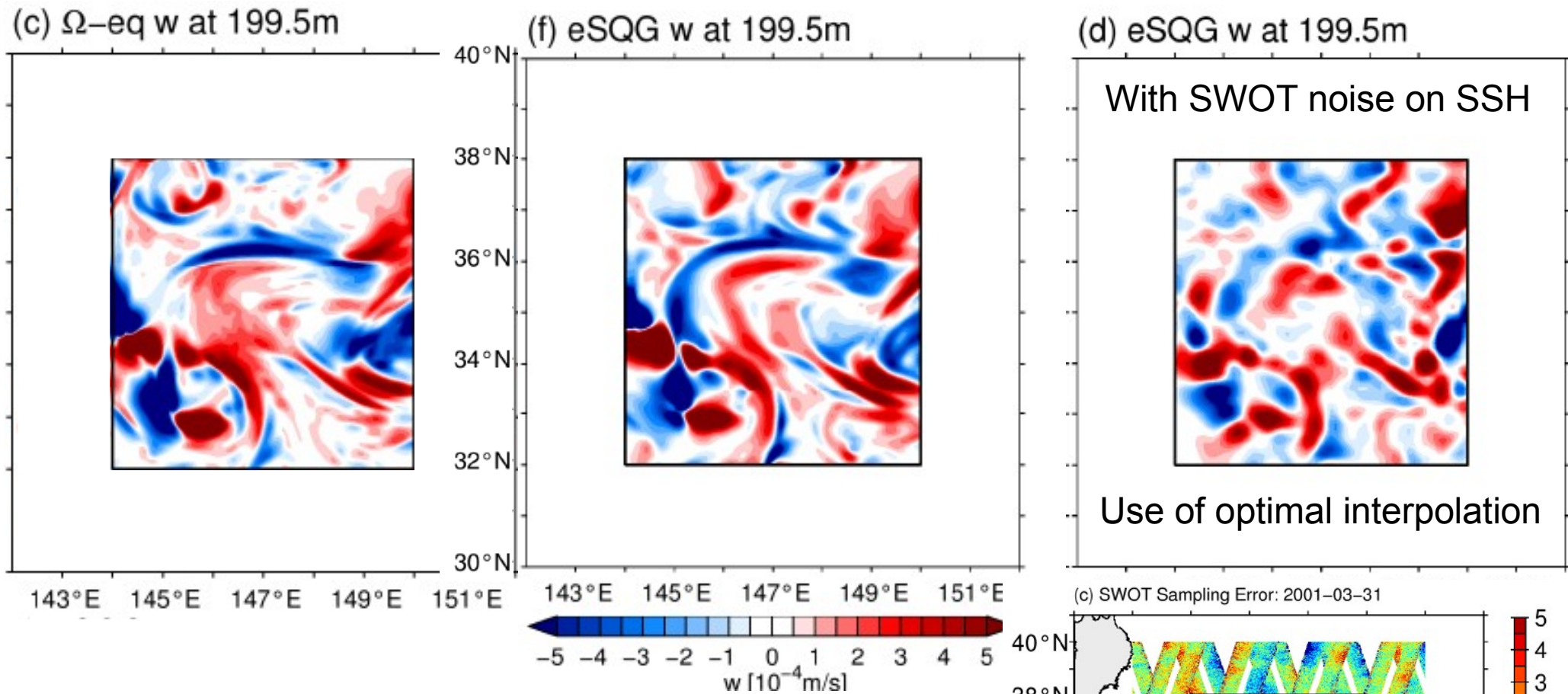


(e) w Mean Spectral Correlation



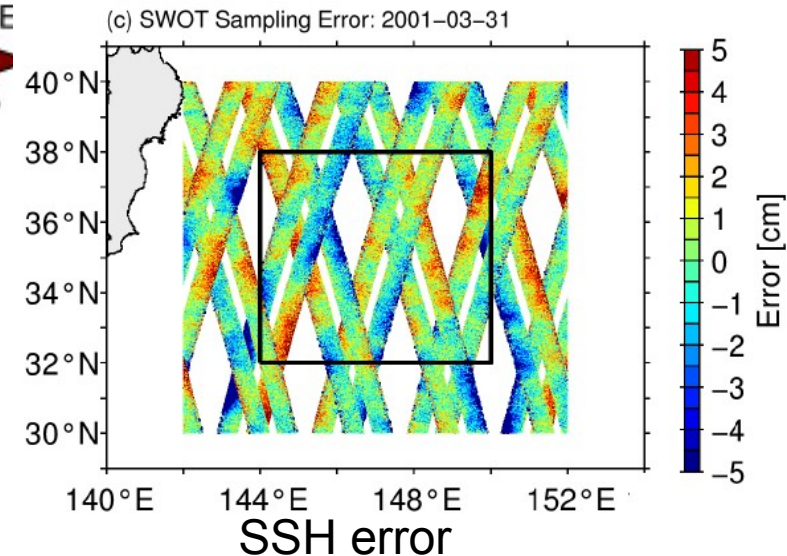
Good reconstructability of w at 200m
but fails in the MXL
and only for "balanced" mesoscales

SQG reconstruction with modeled SSH and no SWOT errors



SQG method still works at mesoscale
when SWOT noise is present

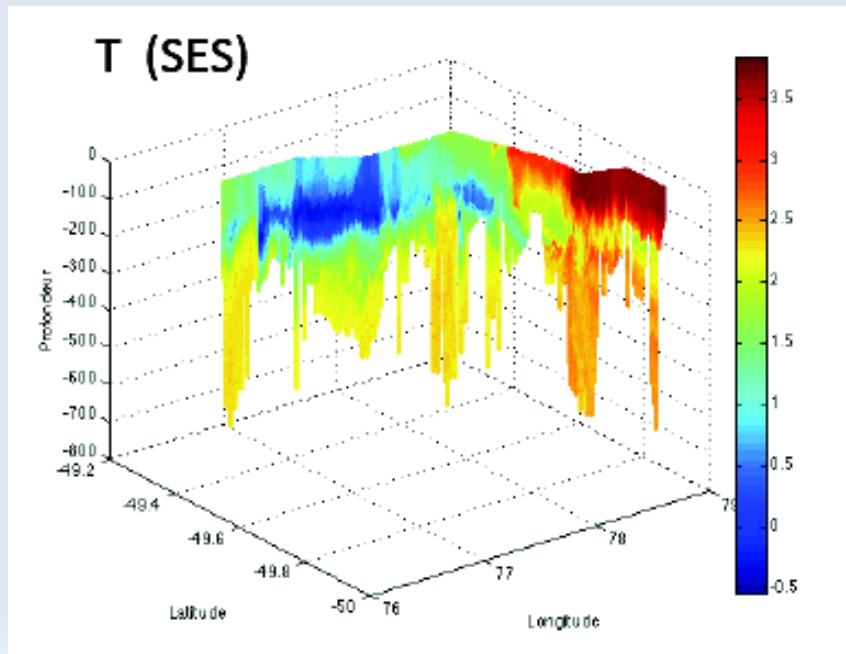
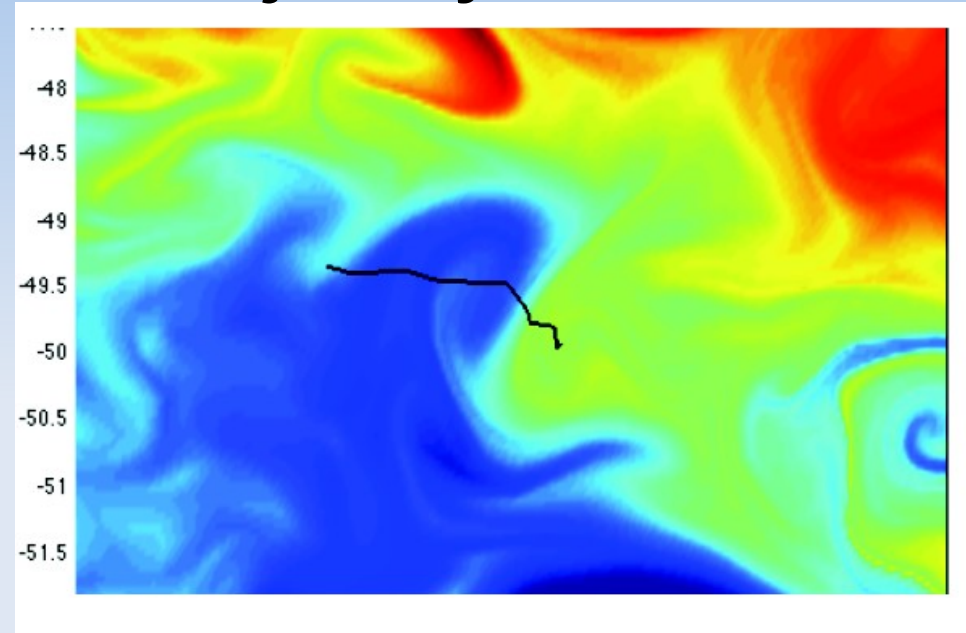
=> use optimal interpolation to
smooth out small-scale signal



Retrieving ocean dynamics using elephant seals



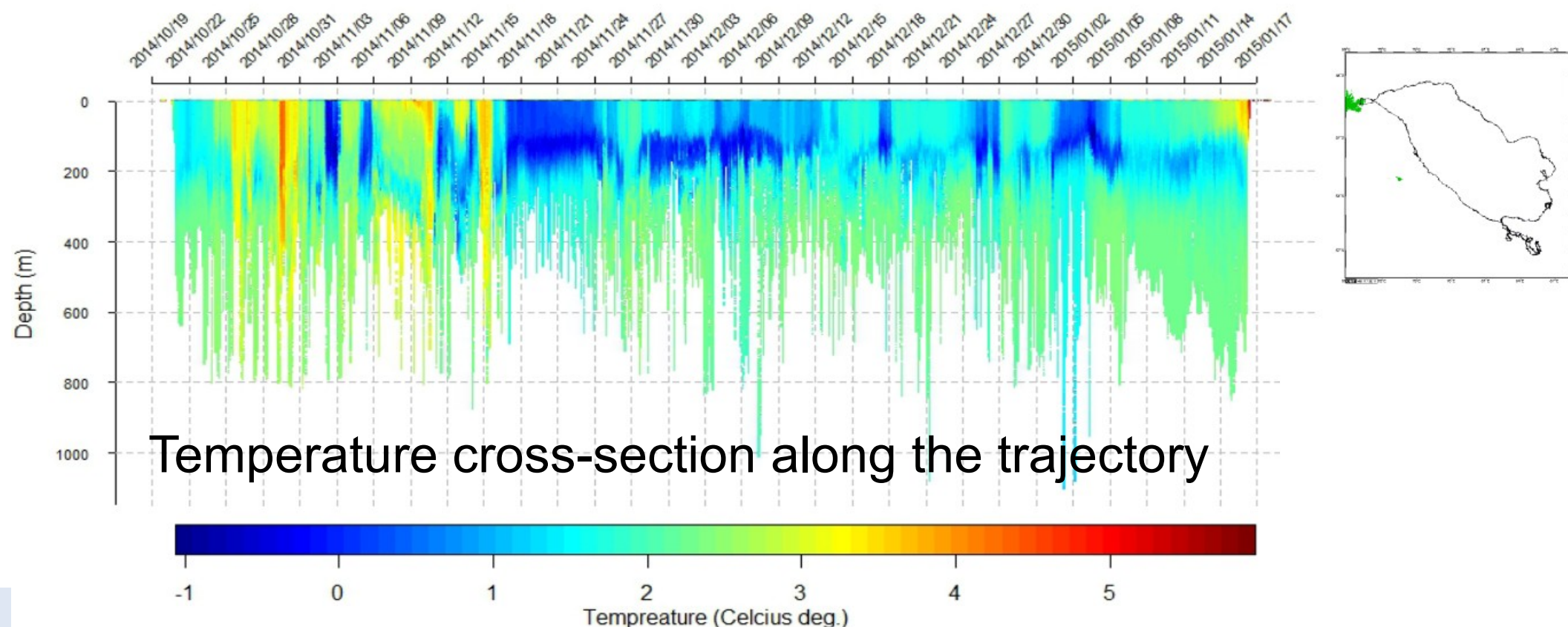
Trajectory and SST



They swim from Kerguelen Island to the ACC and can be tracked by GPS

Provide high-resolution "in situ" T/S with vertical resolution when they dive

Towards vertical velocity reconstruction (P. Rivière, P. Klein, A. Ponte)



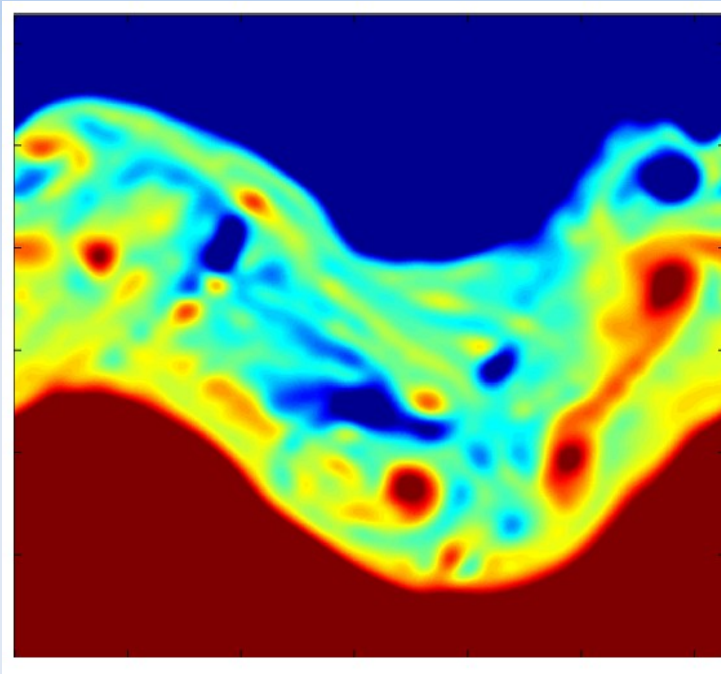
Two approaches will be compared:

- Compute vertical velocities
 - from omega-equation
 - using vertical cross-section of density field (Legal et al. 2007)
- From sQG method with SSH signal

**Thanks
for your attention**

Coupling tracer reconstruction and SQG reconstruction (Berti and Lapeyre)

Typical (microwave) SST from satellite



Smoothed field
but with daily resolution

Taken from a simulation of
idealized turbulence

How to reconstruct submesoscales from smoothed data?

Can we reconstruct both SST and currents?

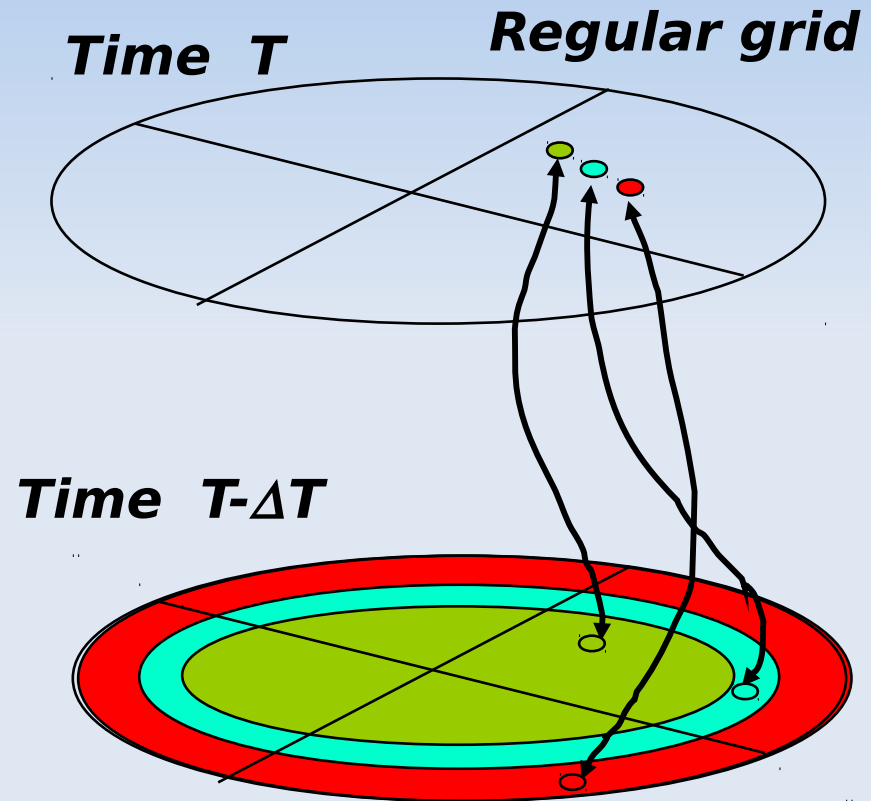
Tracer reconstruction at submesoscale

- **Step 1**

Run a back trajectories starting with a regular gridded array

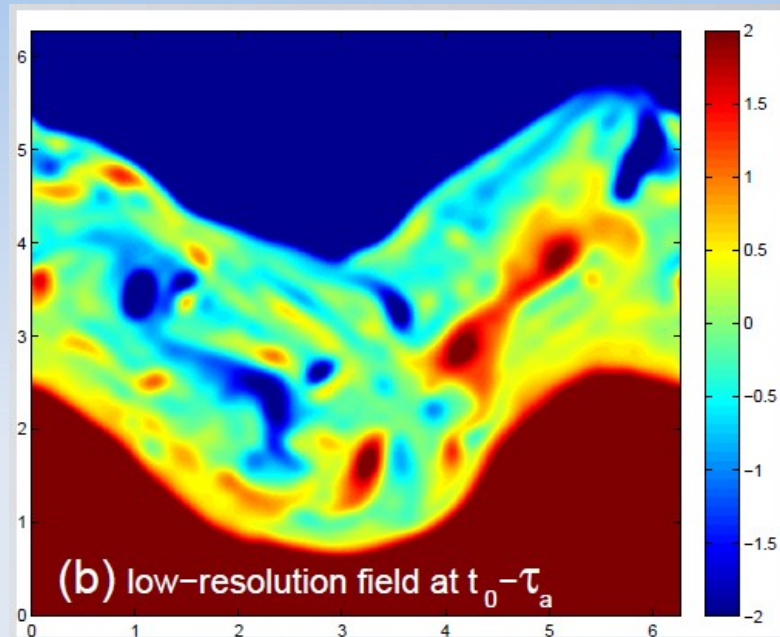
- **Step 2**

Copy values forward to regular grid

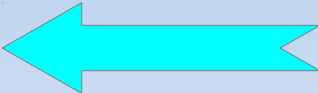


Similar to computation of (FSLE) Lyapunov Exponents
Small scales generated by advection by mesoscale eddies

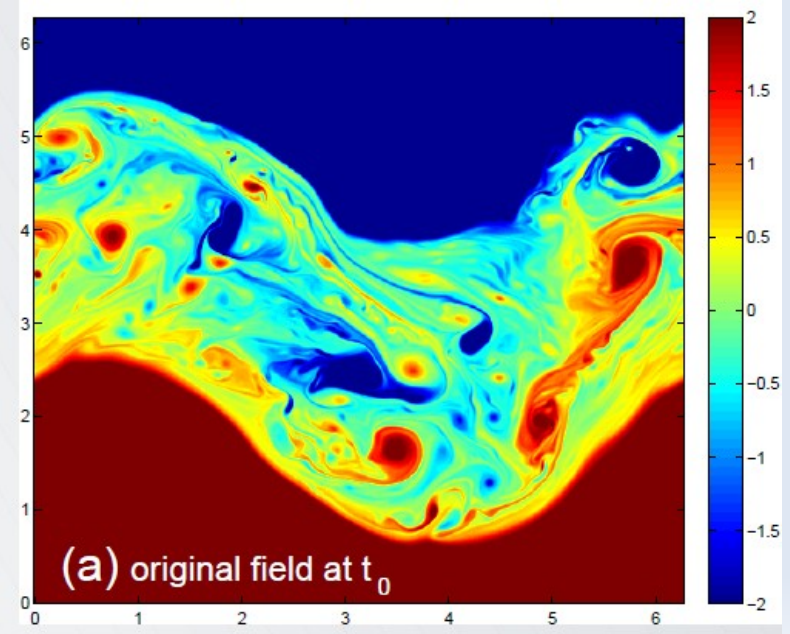
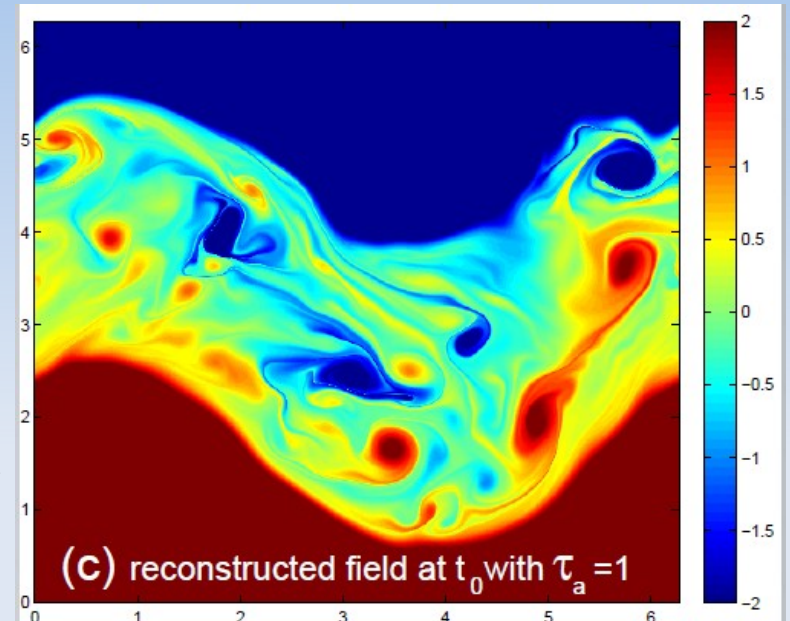

Tracer reconstruction



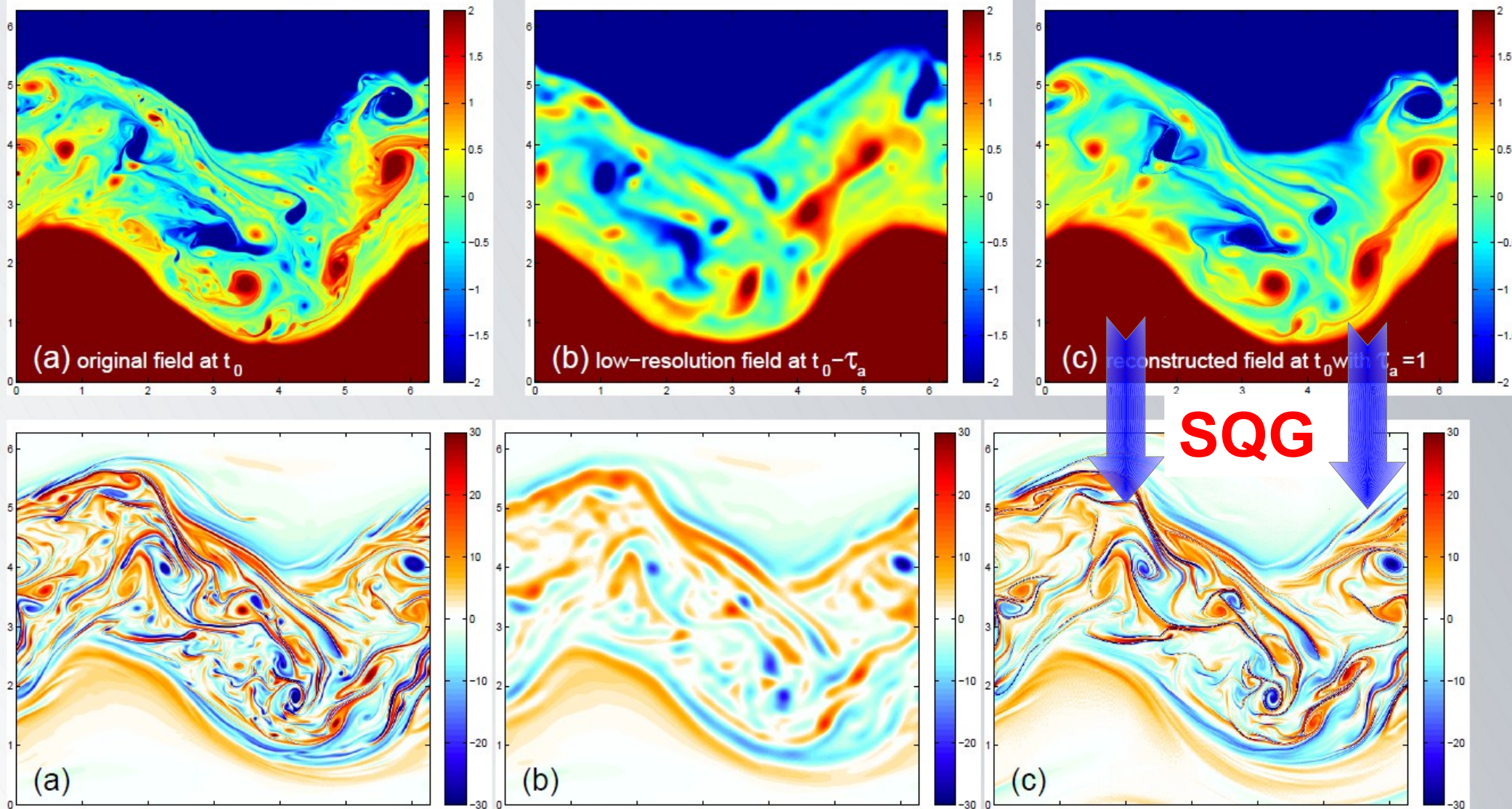
Backward
advection



Tracer
reconstruction



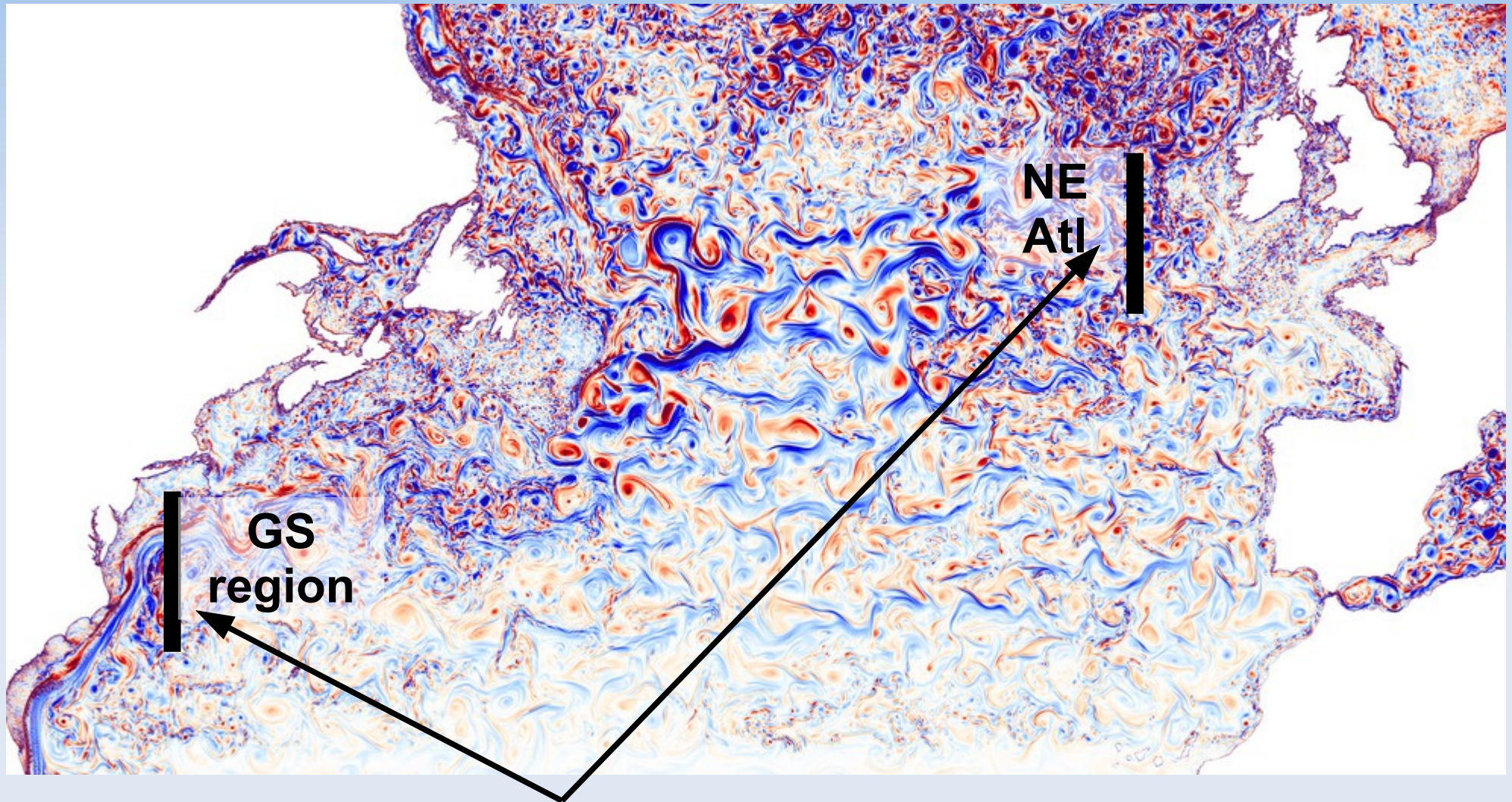
SQG reconstruction: from temperature to velocity



Relative vorticity

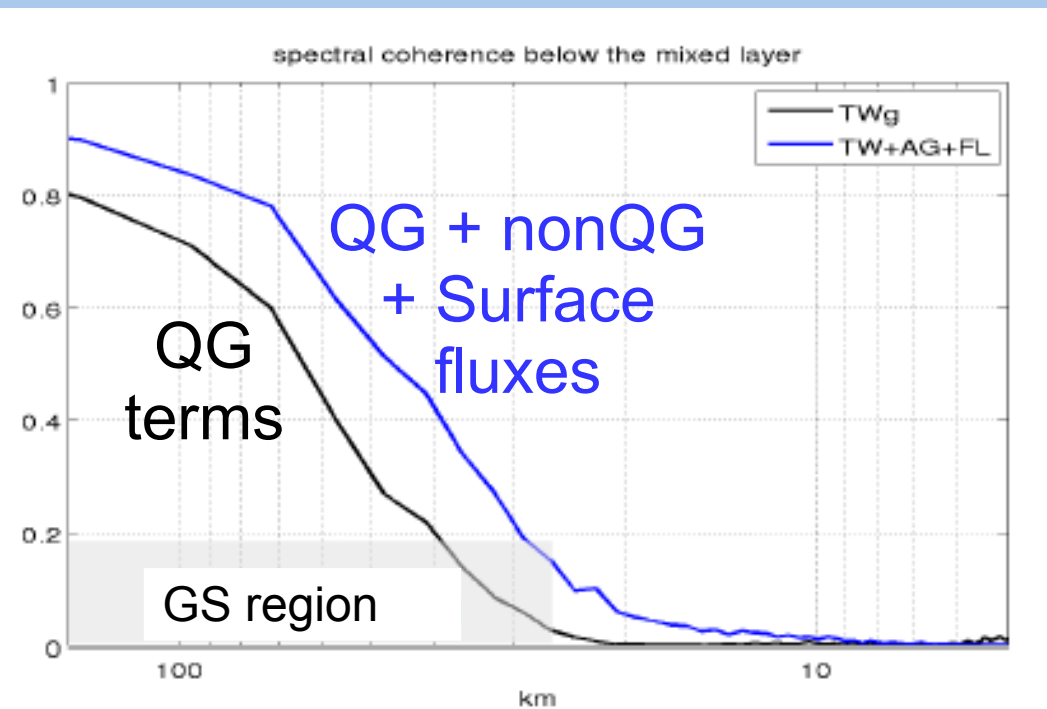
$$\hat{\xi} = k \widehat{SST}$$

North Atlantic simulation (1/60deg) (LGGE team, Grenoble)

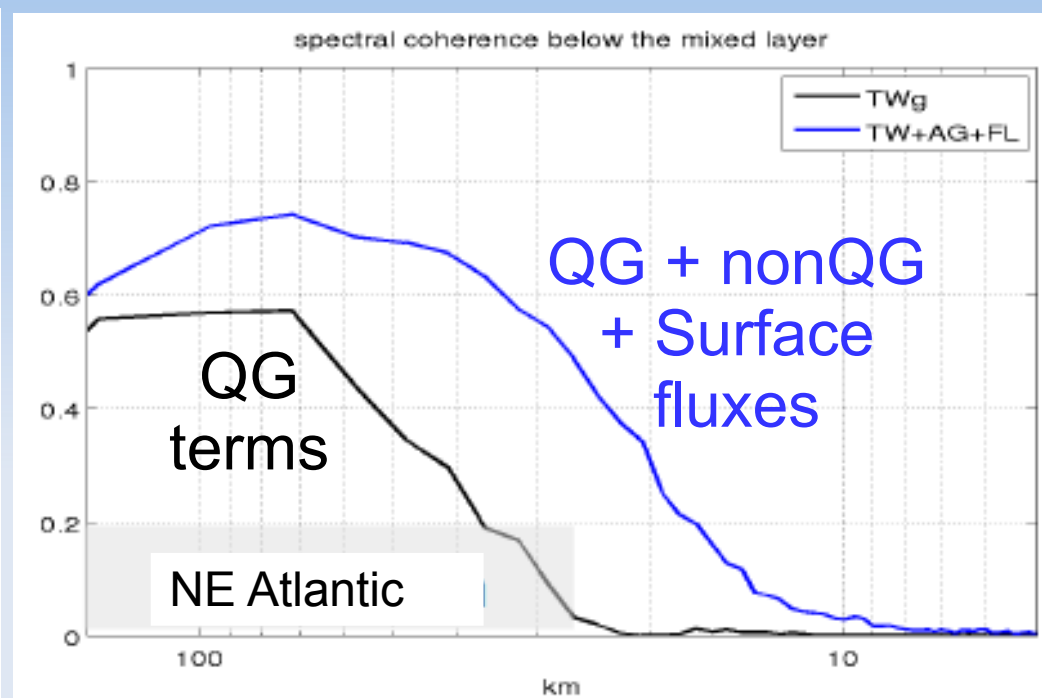


**Vertical cross sections in order to look
at vertical velocities reconstruction**

Validity of reconstruction of w at 300m depth as a function of lengthscale



GS: omega-equation from QG terms does a fair job



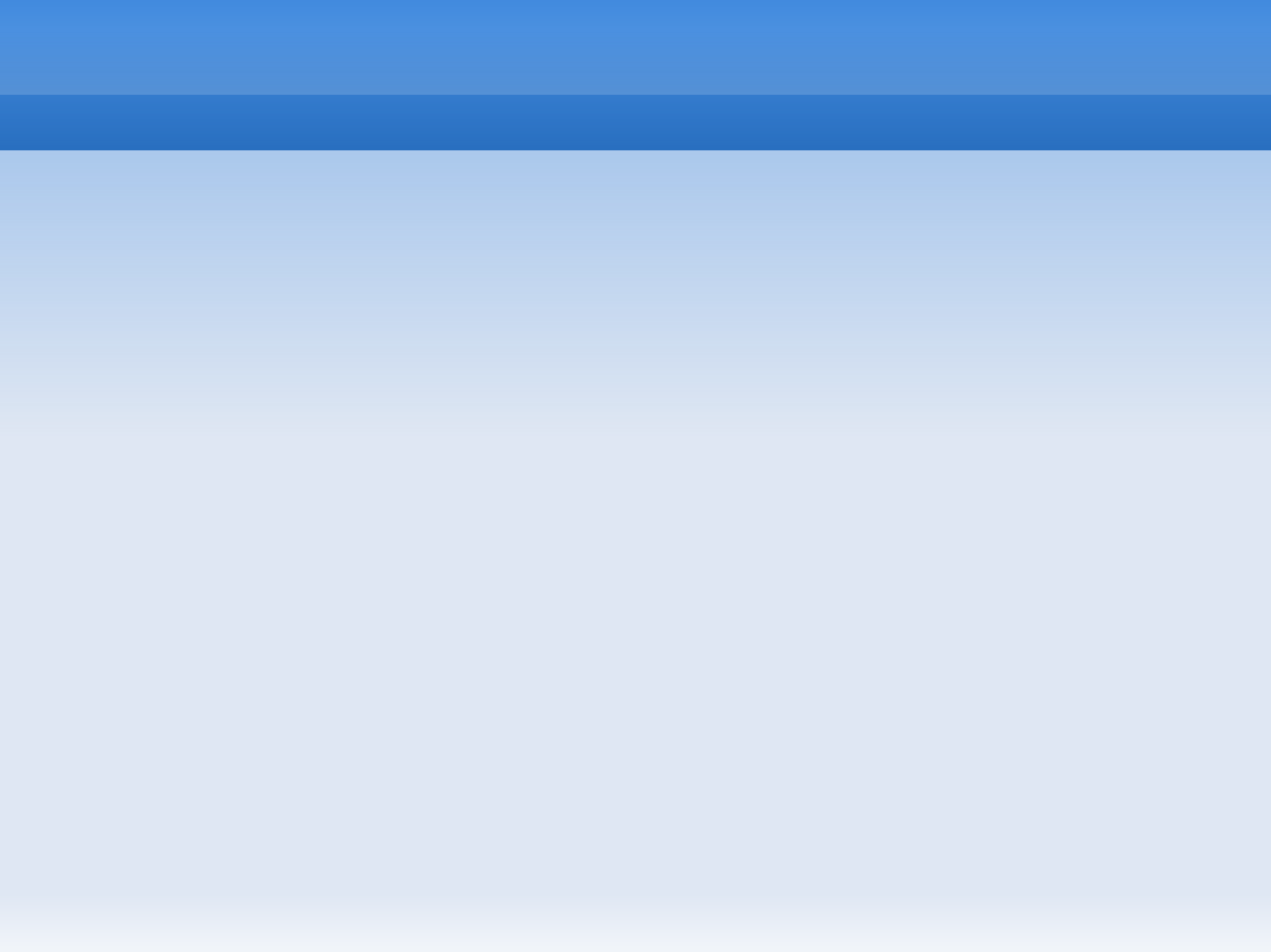
NE Atl: **nongeostrophic** terms are crucial for a good reconstruction

Good reconstructions with

- QG terms only
- adding nongeostrophic terms

only for mesoscales > 60km
up to 30km

Depends on region of study and season



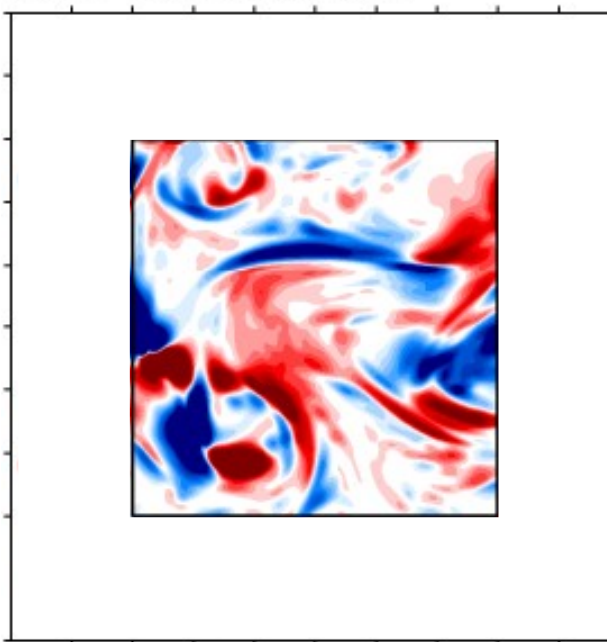
SQG reconstruction with modeled SSH and no SWOT errors

Good reconstructability of w
at 200m but fails in the MXL

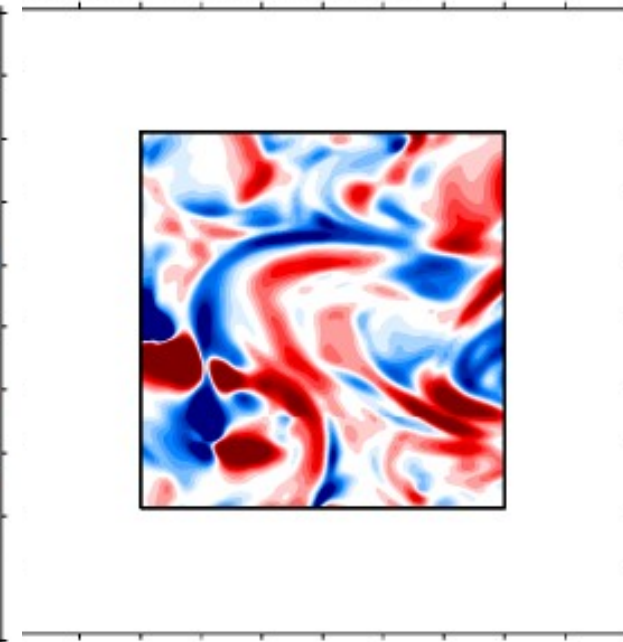
For vorticity, works very well
at all scales down to 200m
and at mesoscales below

For w , works well for
"balanced" mesoscales

(c) Ω -eq w at 199.5m

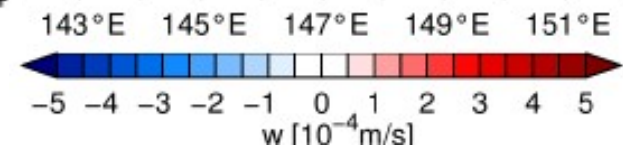
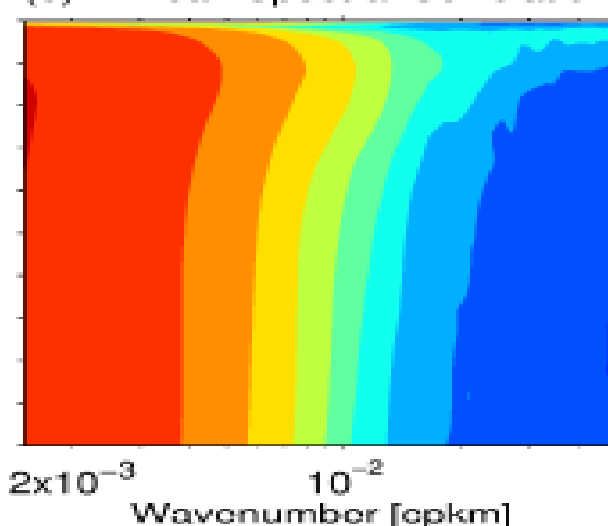
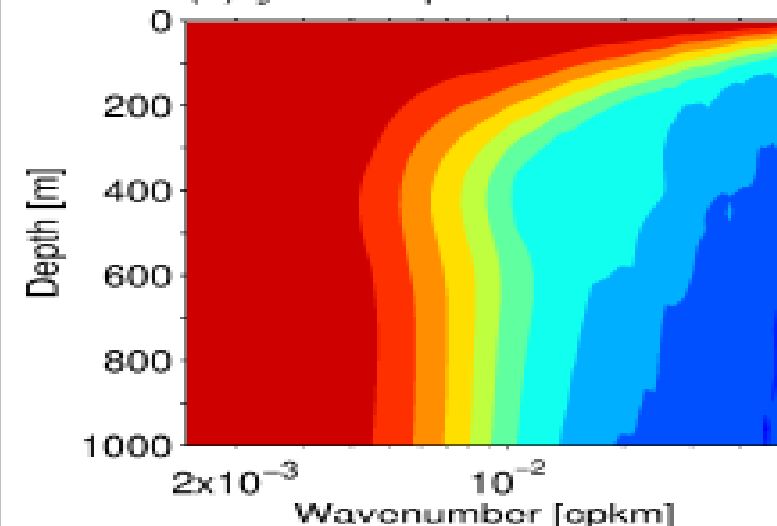


(f) eSQG w at 199.5m

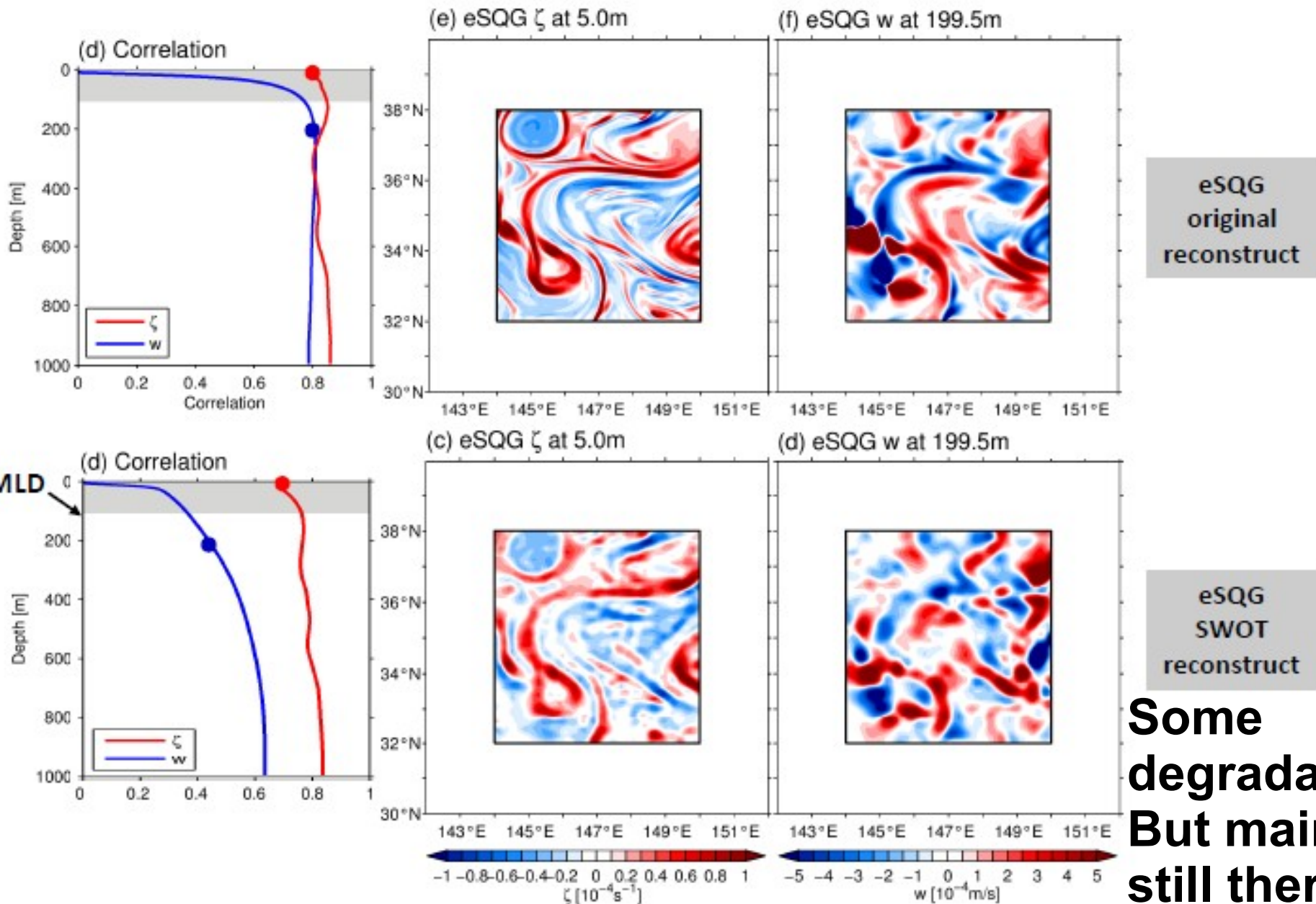


(d) ζ Mean Spectral Correlation

(e) w Mean Spectral Correlation

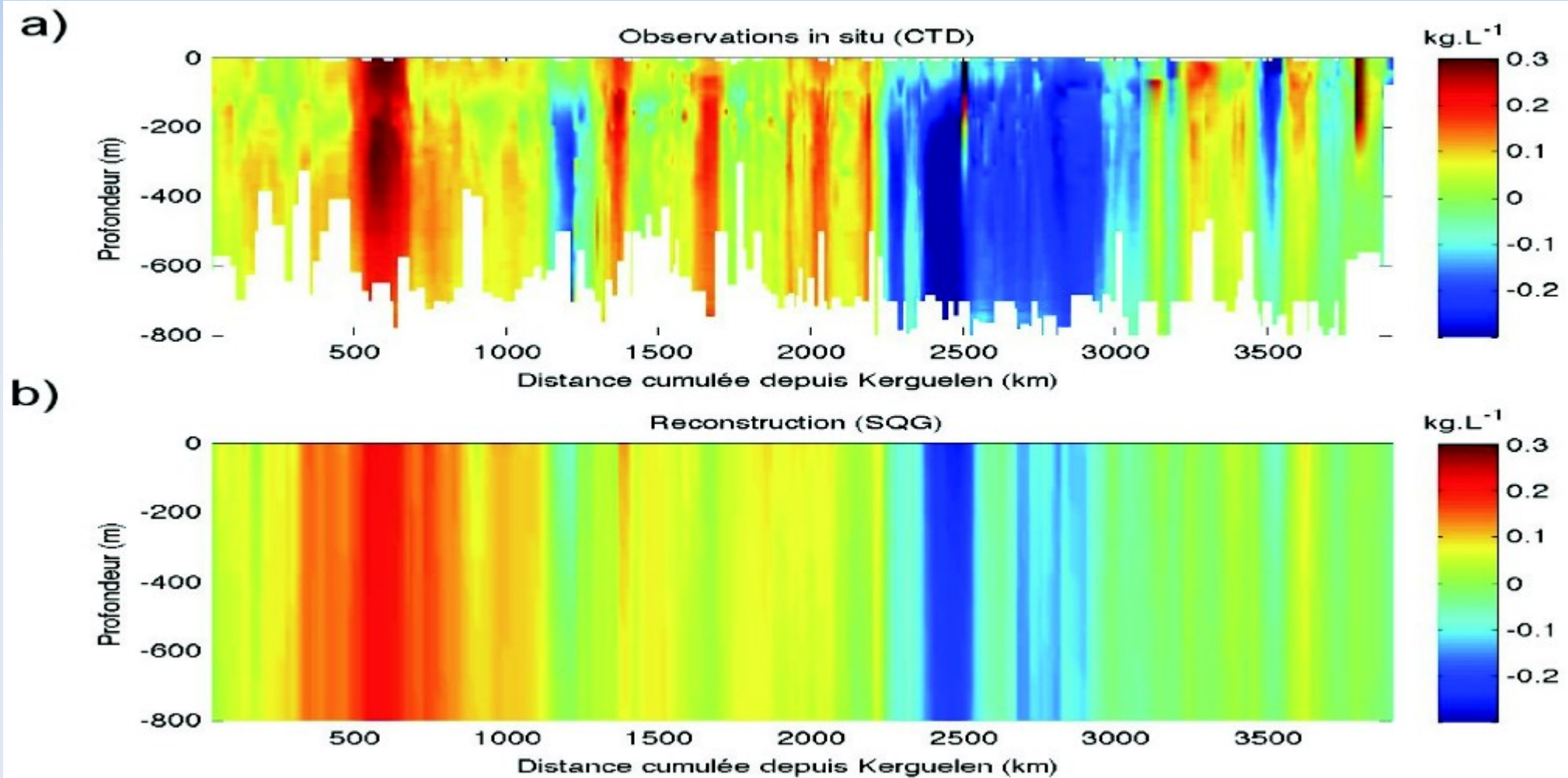


Comparison: SQG reconstruction using noise-free or SWOT simulator SSHs



**Some
degradation
But main signal
still there**

Observed density from "in situ" data



SQG reconstruction using satellite SST